Interpretation of radial correlation reflectometry data 
on Tore Supra and FT-2 tokamaks

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1. Introduction
Radial correlation reflectometry (RCR)\textsuperscript{[1]} is a widely used microwave technique for measuring the properties of electron density fluctuations in tokamaks and more precisely its use is devoted to tokamak turbulence characterization. In this method two frequencies are launched simultaneously into plasma. The coherence decay of two scattering signals with increasing difference of probing frequencies is studied by this diagnostic. The interpretation of RCR results, in general, is a very complicated task even in one dimensional model. Recently a new comprehensive analysis has been performed in 1D model\textsuperscript{[2]} showing the possibility to determine plasma micro-turbulence properties from RCR data. Using this method, we present the results of the proposed theoretical method application to extract turbulence characteristics from experimental data obtained at Tore Supra and FT-2 tokamaks. The radial wave number spectrum and its correlation length as well as spatial turbulence correlation function have been successfully determined. The quality of reconstruction has been proved by numerical simulations for various spectra in conditions relevant to experiment (see also\textsuperscript{[3]}).

2. Basic equations of RCR
Supposing probing wave propagation strictly in the direction of plasma density gradient we use 1D model to treat the RCR problem describing the O-mode probing by equation

\[
\frac{d^2}{dx^2} + \frac{\omega^2}{c^2} n(x) + \frac{4\pi e^2}{m\omega^2} \delta n(x) E_z(x, \omega) = 0
\]

where \( n(x) = n_e x/L \) is the background linear density profile; \( \delta n(x) = \frac{1}{2\pi} \int \delta n e^{-ikx} d\kappa \) stands for turbulent density perturbations assumed statistically homogeneous, \( \kappa \) is a radial wave number; \( \omega \) is a probing frequency; \( E_z \) total field of the probing wave; \( e \) electron charge, \( m_e \) electron mass and \( c \) velocity of light. Under the assumption that density fluctuation level is small \( \delta n(x)/n_e << 1 \) the solution could be found using the perturbation theory methods (Born approximation). The scattering signal amplitude in linear (Born) approximation can be obtained with the help of straightforward approach based upon the reciprocity theorem\textsuperscript{[4]}.

In one-dimensional case it takes the following form: \( A_i(\omega) = \frac{i\omega \sqrt{S_t}}{16\pi} \int\limits_0^L \frac{\delta n(x)}{n_e} E_0^2(x, \omega) dx \), where \( S_t \) is an incident wave energy flux density and \( E_0(x, \omega) \) is a solution of (1) calculated in the absence of density fluctuations that gives the distribution of the probing wave electric field in plasma for \( S_t = 1 \). Finally, as it was shown in\textsuperscript{[2]}, the relation between radial wave number spectrum and RCR CCF takes a form:
where $\Delta L = L_0 - L$ is the cut-off separation; $\text{erf}(s) = \int_0^s e^{-z^2} dz$ is a Fresnel integral and the turbulence wave number spectrum $\tilde{n}_\kappa^2$ is related to the density fluctuation correlation function by expression $2\pi \langle \delta n(x') \delta n(x'') \rangle = \delta n^2 \int_{-\infty}^\infty \tilde{n}_\kappa^2 e^{i\kappa(x'-x)} d\kappa$. The normalized CCF of reflectometry signals is determined as
\[
\overline{\text{CCF}}(\omega_1) = \frac{\left\langle \left( A_\kappa(\omega_0) - \langle A_\kappa(\omega_0) \rangle \right) \left( A_\kappa(\omega_1) - \langle A_\kappa(\omega_1) \rangle \right) \right\rangle}{\left\langle \left| A_\kappa(\omega_0) - \langle A_\kappa(\omega_0) \rangle \right|^2 \right\rangle \left\langle \left| A_\kappa(\omega_1) - \langle A_\kappa(\omega_1) \rangle \right|^2 \right\rangle}
\] (3)
and averaging is held over random phase samples. The turbulence CCF is easily obtained from the reconstructed spectrum (2) using the Fourier transform.

3. RCR simulations and experiment on Tore Supra
Firstly we check the quality of radial wave number spectrum and turbulence CCF (TCCF) reconstruction numerically in conditions close to experiment.

Probing frequency range was chosen according to the settings of the D-band (110-150 GHz) X-mode reflectometer installed at the low field side of Tore Supra tokamak $135 \text{GHz} < f < 145 \text{GHz}$ corresponding to the probing interval $1.85 \text{m} < x < 2.15 \text{m}$, reference frequency $f_0 = 140 \text{GHz}$ corresponds to reference cut-off position $R_0 = 0.85 \text{m}$, see fig. 1. The number of frequencies is $N_\omega = 110$ that corresponds to spatial probing step and frequency step $\delta \Delta = 3 \text{mm}$ of the order $\delta f = 97 \text{MHz}$. The statistical averaging was held over $N_s = 500$ random samples. The input turbulence spectrum is specified as [5] $\delta n^2 = \kappa^{-3} \left( 1 + \kappa^2 \right)^{1/2}$, $|\kappa| \geq 1 \text{cm}^{-1}$; $\delta n^2 = 1$, $|\kappa| < 1 \text{cm}^{-1}$ shown in fig. 2 by red line. The number of wave number modes $N_\kappa = 10000$ is specified within the wave number interval $|\kappa| < 10 \text{cm}^{-1}$. We compute RCR CCF numerically for these parameters in Born approximation according to (3). Further we apply to this simulated RCR CCF the spectrum reconstruction procedure (2). In fig. 2 the reconstructed turbulence spectrum (blue real part) in comparison to the input spectrum (red line) is shown, the complicated $\sim \kappa^{-7}$ dependency is followed by the reconstructed spectrum however the flat part of the spectrum is not reconstructed perfectly. The error level is estimated by imaginary part oscillations and reaches 30% level that could be explained by discontinuities produces by extrapolation procedure and discontinuities in wave number spectrum.
Experiments on Tore Supra were the first attempt to apply the theoretical method [2] of turbulence properties determination at real machine. RCR measurements have been held at Tore Supra tokamak (shot #47669) during summer 2011 campaign using the D-band of the X-mode reflectometer installed from the low field side of the torus. Plasma density profile \( t=7s, f_0=138GHz, R_0 = 2.024m \) is shown in fig. 1. Black vertical lines show the spatial probing interval \( \Lambda \) which is situated at the high field side of the plasma in the close to linear plasma density profile behavior region. Exponentially growing distance between the cut-off positions was chosen for the experiment (20 points per \(-25cm\), corresponding to \( f = 138 \pm 5GHz \)).

![Fig. 3. Reconstructed turbulence wave number spectrum. \( \Omega = 100kHZ \)][3]

The turbulence radial wave number spectrum was reconstructed according to (2) in the range of turbulence frequencies \( \Delta \Omega = 1MHz \) after the extrapolation procedure at \( \Delta R = 0.02m \). The spectrum reconstructed from the experiment agrees with the behavior \( n^2_k \propto k^{-3} \) for \( 5cm^{-1} < |k| < 10cm^{-1} \), the spectrum growth at small wave numbers saturates for \( 1cm^{-1} < |k| < 5cm^{-1} \) and shows slow decay for \( |k| < 1cm^{-1} \), within \( \delta k = 0.04cm^{-1} \) (see fig. 3). TCCF (red line in fig. 4) was computed as a Fourier transform of the spectrum and its correlation length was estimated rather low, as \( l_c \approx 2mm \) at R=2.024m.

4. RCR simulations and experiment on FT-2

We have simulated the X-mode RCR experiment on FT-2 using the following parameters close to experiment: \( f_0 = 40GHz, 21GHz < f < 49GHz \quad \delta f = 0.1GHz; \quad n^2_k = \exp(-k^2l_c^2/4) \), \( l_c = 5mm \) (see fig. 6, red line). Plasma density profile taken from the experiment is shown in fig. 5. The reconstructed spectrum (black line real part, green line imaginary part) is shown in fig. 6.

![Fig. 4. Signal RCR CCF (blue) and TCCF (red). \( \Omega = 100kHZ \)][4]

The signal CCF has been measured in experiment at the FT-2 tokamak \( B_t = 1.8T, I_p = 19kA, T_e = 500eV, n_s(0) = 3 \cdot 10^{13}cm^{-3} \) using X-mode reflectometer from the high field side of the torus (see Fig. 5), equidistant probing step was chosen (50 points per \(-3cm\), corresponding to \( f_0 \pm 3GHz, f_0 = 60GHz \), parameters of probing window: \( N_w = 50, N_z = 155, N_\omega = 64 \)), signal wave frequency step \( \delta f \) varied from 0.025GHz (in the vicinity of reference cut-off position) to 0.1GHz. The double antenna set was shifted out of equatorial plane by 15mm that corresponds to poloidal probing wave number of \( 3cm^{-1} \). The corresponding reflectometry (Doppler) spectrum is shifted by 400 kHz from the probing frequency. A very wide (lasting without decay till the Bragg limit for reflectometry equal to \( 22cm^{-1} \)) radial wave number spectrum (real part) is shown in fig. 7. The width of this spectrum is growing with the turbulence frequency demonstrating a clear dispersion. The accuracy of spectrum reconstruction (estimated using the spectrum imaginary part and negative bursts of real part) is not high, however sufficient for estimating the spectrum width from the bottom. It corresponds to the turbulence correlation length substantially smaller than \( 5mm \). The suppression of the spectrum at very small wave numbers is most likely related to the limitations of the 1D model. The turbulent density...
fluctuation two-point correlation function is provided by the Fourier transform of the experimentally obtained spectrum in which only the contribution of physically sensible part within the Bragg limit is accounted for. The result of this transform is shown in fig. 8, demonstrating decrease of the correlation length with growing frequency.

It is important to underline that all the results presented here are obtained at small tokamak FT-2 where small radius is \( a=8\,\text{cm} \) and probing wavelength is \( \lambda = 4\,\text{mm} \) and the relation \( L_{\|} f_0 / c = 2 \cdot 10 \). Due to small size of the device the 2D effects can make impact on the resulting signal RCR CCF, moreover the multiple reflections between cut-off and antennae take place in case of equatorial plane probing and spoil the resulting spectrum and TCCF.

**5. Conclusion**

Reflectometry signals have been correctly described analytically in the frame of Born approximation [2]. Capability of the method has been evaluated in numerical modeling. We have applied the method at Tore Supra and FT-2 tokamaks and the first positive results have been obtained.

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