Schémas d'ordre élevés pour la simulation de la turbulence et des instabilités dans les plasmas de bord de tokamaks

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Colloque de prospective de la FR-FCM, jeudi 24 novembre.

- Development of **advanced numerical methods**
  - high order schemes
    $\Longrightarrow$ well adapted to the simulation of instabilities (or turbulence).
  - on unstructured meshes
    $\Longrightarrow$ able to handle complex geometries.

- Simulation of edge plasma in tokamaks:
  - in toroidal geometry.
  - multi-fluid full braginskii model coupled with Maxwell equations

    $\rightsquigarrow$ bi-fluid full Braginskii model with the electrostatic assumption.

    $\rightsquigarrow$ (reduced) MHD model.

- Development of computational tools

FINITE VOLUME METHODS FOR MHD MODELS

E. Estibals (PhD), H. Guillard, A. Sangam

- Projection at each time step on a rotated gradient field
  - strict respect of the divergence free constraint on the magnetic field
  - extremely cheap

$\mathcal{C}^1$ FINITE ELEMENT METHODS FOR MHD MODELS

J. Costa (PhD), H. Guillard, B. Nkonga

- Introduction of a vector potential
  - efficient way to deal with divergence free constraint on the magnetic field
  - introduce of higher order derivatives terms
- Two strategies
  - quadrangular cubic Hermite Bezier basis
  - 2D triangular Powell Sabin FE

$\mathcal{C}^0$ FINITE ELEMENT METHODS FOR FULL BRAGINSKII MODELS

S. Minjeaud, R. Pasquetti

- Spectral Element Method

- Mixing of finite elements and spectral methods
  - Handle complex geometries,
  - Well adapted to non-linear and anisotropic PDEs,
  - Spectrally accurate wrt the polynomial approx. degree (for smooth pb).
- Based on a weak formulation of the PDEs and a Galerkin approx.
  - Reference (master) element: $Q_{ref} = [-1, 1] \times [-1, 1]$.
  - P_N approximation on Q_{ref}.
  - Interpolation / quadrature points : tensorial product of the GLL points.
  - Basis functions: Lagrange polynomials based on the GLL points.
  - Quadratures exact in P_{2N-1}.
  - Isoparametric mappings, $\mathbf{f}_Q : Q_{ref} \to Q$.

- Geometry:
  - 3D geometry with a periodic direction (toroidal domains)
  - high flexibility with the poloidal geometry (unstructured meshes)
- Numerical schemes: high order methods
  - Fourier approx. in the toroidal direction (uniform mesh)
  - SEM in poloidal planes (2D unstructured quadrangular mesh)
- Linear Algebra:
  - native matrix free solver (cg or bicgstab) with jacobi precond.,
  - coupling with PETSC library (large choice of precond.)
- a Fortran-MPI implementation:
  - domain decomposition of the poloidal plane,
  - distribution of the poloidal planes and Fourier modes,
  - Almost transparent for the user.
- Model:
  - provides high level routines to assemble the different terms,
  - Spectral Vanishing Viscosity stabilization technique.

MODELING HYPOTHESIS

- A two-fluid ion-electron modeling,
- Conservation of density, momentum and energy + Braginskii closures,
- Electric neutrality: $\sum_s n_s e_s = 0$ (n_s: density, e_s: charge),
- $\boldsymbol{B}$ is fixed and axisymmetric,
- $\boldsymbol{E}$ is electrostatic: $\boldsymbol{E} = -\nabla U$,
- State laws $p_s = n_s T_s$ and $\varepsilon_s = p_s/(\gamma - 1)$.

THE FULL PDES SYSTEMS

- $\rho_s = n_s m_s$, $\rho = \sum_s \rho_s$ and $\boldsymbol{q}_s = \rho_s \boldsymbol{u}_s$.
- $w_s = e_s/m_s$ and $\alpha_s = w_s \rho_s / \rho$

$$\begin{aligned}
&\partial_t \rho + \nabla \cdot (\boldsymbol{q}_i + \boldsymbol{q}_e) = 0 \\
&\partial_t \boldsymbol{q}_s + \nabla \cdot (\boldsymbol{q}_s \boldsymbol{u}_s + p_s I + \Pi_s) = -\alpha_s \rho \nabla U + w_s \boldsymbol{q}_s \wedge \boldsymbol{B} + \boldsymbol{R}_s \\
&\nabla \cdot (w_i \boldsymbol{q}_i + w_e \boldsymbol{q}_e) = 0 \\
&\partial_t \varepsilon_s + \nabla \cdot (\varepsilon_s \boldsymbol{u}_s + \boldsymbol{\varphi}_s) = -p_s \nabla \cdot \boldsymbol{u}_s - \Pi_s : \nabla \boldsymbol{u}_s + Q_s
\end{aligned}$$

$\rightsquigarrow$ 10 scalar PDEs,
$\rightsquigarrow$ 10 unknown scalar fields: ρ (1), $\boldsymbol{q}_s$ (6), ε_s (2), U (1)

STRANG SPLITTING

▶ System I: explicit treatment (ERK scheme) and subtime cycling

$$
\begin{aligned}
&\partial_t \rho + \nabla \cdot (\boldsymbol{q}_i + \boldsymbol{q}_e) = 0 \\
&\partial_t \boldsymbol{q}_s + \nabla \cdot (\boldsymbol{q}_s \boldsymbol{u}_s + p_s I + \Pi_s) = \boldsymbol{R}_s \\
&\partial_t p_s + \nabla \cdot (p_s \boldsymbol{u}_s + (\gamma - 1)\boldsymbol{\varphi}_s) = (\gamma - 1)(-p_s \nabla \cdot \boldsymbol{u}_s - \Pi_s : \nabla \boldsymbol{u}_s + Q_s)
\end{aligned}
$$

▶ System II: implicit treatment (DIRK scheme)

$$
\begin{aligned}
\partial_t \boldsymbol{q}_s &= -\alpha_s \rho \nabla U + w_s \boldsymbol{q}_s \wedge \boldsymbol{B} \\
\nabla \cdot (w_i \boldsymbol{q}_i &+ w_e \boldsymbol{q}_e) = 0
\end{aligned}
$$

EQUATION FOR THE POTENTIAL

▶ Elimination of the $\boldsymbol{q}_s$ in System II

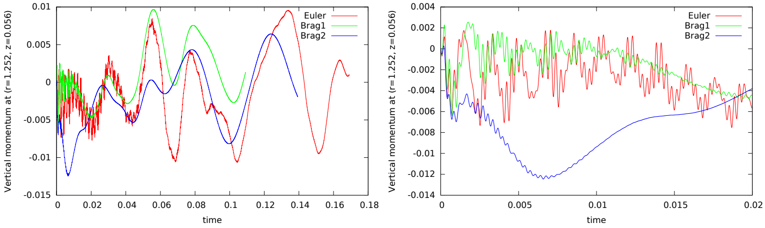
$$
\nabla \cdot A \nabla U = \sum_s w_s \nabla \cdot A_s \tilde{\boldsymbol{q}}_s
$$

where $\tilde{\boldsymbol{q}}_s$ is the previous ERK estimate of $\boldsymbol{q}_s$ and

- A is definite positive
- A is an anisotropic tensor, the anisotropy ratio depends on δt

	$\delta t = \omega_{ci}$	$\delta t = \omega_{ce}$
$A_\perp / A_\parallel$	$5.01\ 10^{-5}$	$9.24\ 10^{-2}$

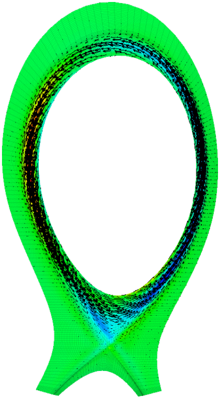
- Numerical experiments



Vertical momentum at $(1.25, 0.056)$ for the Euler and for two Braginskii computations

- Observation: **dominant frequencies** $\approx 3000, 40$.
- Link with cyclotronic frequencies ?
  - At this point, in dimensionless form $B = 2375$,
  - $f_{ic} = Be/(2\pi m_i) \approx 378$ and $f_{ec} = Be/(2\pi m_e) = 100 f_{ic} \approx 37800$

$$Be/(2\pi\sqrt{m_e m_i}) \approx 3780 \rightsquigarrow \text{same order of magnitude...}$$



current

- Preliminary computations :
  - 31194 grid points (3000 cells with $n = 3$).
  - Computations up to few micro-seconds.
  - Heavy electron assumption : $m_e = m_i/100$
  - Parallel transport coefficients divided by 10
- Difficulties :
  - Bohm boundary conditions
  - Slip BC at the center of the domain ?

S. Minjeaud, R. Pasquetti, JCP, 2016

SEM scheme for a reduced MHD model

H. Guillard, S. Minjeaud, R. Pasquetti

$$\begin{aligned}
&\partial_t \mathbf{u} + (\mathbf{u}\cdot\nabla)\mathbf{u} + \nabla\lambda = (\mathbf{b}\cdot\nabla)\mathbf{b} + \partial_z \mathbf{b},\\
&\partial_t \mathbf{b} + (\mathbf{u}\cdot\nabla)\mathbf{b} = (\mathbf{b}\cdot\nabla)\mathbf{u} + \partial_z \mathbf{u},\\
&\nabla\cdot\mathbf{u} = 0.
\end{aligned}$$

H. Guillard, INRIA Research report, 15.

- Comparison with other codes.
- Cross validation of the numerical strategies.
- Comparison accuracy/cost.